Spin Waves_____

Localized Defect Modes in a Two-Dimensional Array of Magnetic Nanodots

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Abstract—Microwave properties of an array of magnetic nanodots in a ferromagnetic state having a point defect – a dot with inverted magnetization or different material parameters - are considered. The existence of a single point defect in a dot array may lead to the appearance of several localized modes: one "defect" eigenmode and several "well" modes, the number and structure of which strongly depend on the magnetostatic interaction between the dots. It is shown that by performing a ferromagnetic resonance (FMR) experiment in an array of magnetic dots containing a small number of defects it is possible to obtain the information about the entire spin-wave spectrum of the array.

I. INTRODUCTION

Arrays of magnetic nano-dots are promising candidates for applications in microwave signal processing and in magnetic data recording technology [Sellmyer 2006]. Modern patterning technologies allow one to fabricate dot arrays with interdot distances that are sufficiently small to guarantee a significant magnetostatic interaction between the dots. This interaction leads to the dependence of the array's properties on the static magnetic configuration (or metastable stationary state) of an array [Bondarenko 2010, Verba 2012]. The metastable stationary state of an array is not unique – there may be several stable states corresponding to the same external conditions. This multistability opens a way for the development of a novel type of artificial materials with tunable microwave properties – reconfigurable magnonic crystals (RMC) [Tacchi 2010, Topp 2010, Verba 2012].

Since the real-life dot arrays are not perfect, it is of a great practical importance to investigate the influence of different kind of defects on the properties of RMC. In general, defects lead to such undesirable effects as broadening of the ferromagnetic resonance (FMR), random scattering of spin waves (SWs), etc. [Gurevich 1996]. Also defects can cause formation of defect bands in prohibited SW zones [Kruglyak 2006], which may possibly have their own applications, analogous to various applications of defects in photonic crystals [Sakoda 2001].

The defects in RMC can be of a various nature. First, there can be common ("material") defects caused by the technological imperfections, such as spread of geometrical and/or material parameters of the dots. Then, the multistability of the array's stationary state leads to a second kind of defects – defects of the magnetization order - that are breaking the ideal magnetic periodicity in an array. Such defects can be formed by a single dot or a small group of dots having inverted direction of static magnetization (in respect to the majority of dots in the array). These defects of the magnetic order could not be eliminated and are, in principle, not small. Thus, they could substantially affect the array's microwave properties.

In our present Letter we consider the influence of a single "point" defect (a single defect dot in an array) of both kinds on the microwave properties of an array existing in a ferromagnetic (FM) metastable stationary state, in which the magnetizations of all the dots are parallel. We considered an array of *axially* magnetized cylindrical nanodots arranged in a square lattice (shown in Fig. 1a). All the numerical calculations presented below are made in a macrospin approximation using the method described in [Verba 2012] for the following array's geometry: dot aspect ratio $h/R = 5$, lattice constant $a = 4R$. To avoid the influence of the edge effects, we used periodic boundary conditions at the array's edges, so, formally, a periodic FM state of an array with periodic defects was considered. However, if the separation between the defects is sufficiently large the interaction between them could be considered negligible.

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II. DEFECT OF THE MAGNETIZATION ORDER

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14. ABSTRACT
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15. SUBJECT TERMS
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First, let us consider a point defect of the magnetization order. For a FM stationary state this is a single dot with inverted direction of magnetization (Fig. 1a). The stationary metastable states with such defects are stable in a wide range of bias magnetic field and dots' geometric parameters. For example, in a zero bias field such "defect" states are more stable than the ideal FM state, since they correspond to a smaller value of the array's total magnetic moment.

In the majority of standard microwave experiments and in practical applications the characteristic scale of the spatial nonuniformity of external microwave fields is, usually, much larger than the size of the dot array, and, therefore, the external microwave fields could be considered spatially uniform. The spectrum of a spatially uniform ferromagnetic resonance (FMR) of a dot array in the ideal periodic FM stationary state has one absorption peak at the frequency

$$\omega_{FMR} = \gamma B_e + \omega_M \left(F_0^{xx} - F_0^{zz} \right), \qquad (1)$$

where γ is the gyromagnetic ratio, B_e is an external magnetic field, directed along the magnetization of dots (in z-direction), $\omega_M = \gamma \mu_0 M_s$ and $\hat{F}_k$ is array's demagnetization tensor [Verba 2012].

FIG 1 HERE

The FMR absorption spectrum of a dot array containing an isolated defect of a magnetization order in the FM stationary state is shown in Fig. 1a. The presence of a defect only weakly changes the spectrum of the "volume" SW modes (VMs) in an array: the mode's frequencies simply increase slightly due to the static demagnetization field of a single dot forming a point defect. This frequency shift is proportional to the density of defects (ratio of the number of the "defect" dots n_{def} to the total number n_d of dots in an array, and is negligible if $n_{def}/n_d < \alpha_G$, since the Gilbert damping constant α_G determines the width of the FMR resonance line in an ideal periodic state. Also, the defect creates a certain spatial non-uniformity in the profiles of the VMs [Verba 2012]. Due to this defect-related spatial non-uniformity VMs with nonzero wave vectors could be excited by a uniform microwave field (see many small peaks in the VM (green) range in Fig. 1b).

Another significant absorption peak in FMR the spectrum of the array seen at a higher frequency is related to the excitation of a "defect" SW mode (DM). This mode is mainly localized on the "defect" dot (see Fig. 1c), and has a polarization determined by the defect (left circular for our geometry, while the VMs are mainly right circular polarized). Due to the different polarizations of the DM and the VMs one can neglect dynamical interaction between them, and obtain the following expression for the DM frequency

$$\omega_{DM} = -\gamma B_e + \omega_M \left(F_0^{zz} + N_s^{xx} - 2N_s^{zz} \right), \qquad (2)$$

where $\hat{N}_s$ is the self-demagnetization tensor of dot [Gurevich 1996]. Obviously, in a zero external field the DM has a larger frequency due to the larger static internal field at the defect. At the fields $B > B_c = \mu_0 M_s \left(F_0^{zz} + N_s^{xx} - 2N_s^{zz} \right)$ the frequency of the DM becomes negative, which corresponds to the instability of the stationary state with a defect. For such fields only the ideal FM stationary state remains stable.

However, as one can see from Fig. 1a, there are also other absorption peaks outside of the "volume" SW spectrum of the array. The SW modes corresponding to these peaks are also localized in the vicinity of the defect, but, in a drastic contrast with the conventional DM, these modes have vanishingly small amplitudes at the defect dot itself (see e.g. Fig. 1d). The physical reason for the appearance of these additional localized modes, which can be called the "well" modes (WMs) [Jorzick 2002], is the local inhomogeneity of the dipolar field (in our case – the local maximum of this field) caused by the inverted magnetization of the defect dot. The small amplitudes of the WMs at the location of the defect dot are caused by the different polarizations of the magnetization oscillations in the defect dot and in all the other dots. The structure of the WMs is very sensitive to the strength of the magnetodipolar interaction between the dots. Besides the WM shown in Fig. 1d, there are also anti-symmetric WMs with similar profiles and, also, higher WMs with more complicated spatial profiles. The total number of WMs increases with the increase of the defect-related potential well depth (which increases with the increase of the zz-component of the mutual demagnetization tensor of dots $\hat{N}(r)$ [Beleggia 2004]), but decreases when the dynamic dipolar interaction (xx, xy and yy components of $\hat{N}(r)$) becomes stronger. Therefore, by studying the WMs spectra one can gain valuable information about the dipolar coupling between the dots in a dots array.

FIG 2 HERE

According to (1) and (2), the fundamental VM and DM have the opposite dependences on the bias magnetic field B_e. Thus, the "defect" mode intersects with the array's volume SW spectrum at a certain magnetic field (see Fig. 2a, $B_e \approx 0.14 \mu_0 M_s$). Near this point the DM hybridizes with the closest SW mode, which is not orthogonal to the DM. At lower fields, when the DM approaches the upper limit of the SW spectrum, it hybridizes with the highest symmetric WM. At higher fields, while approaching the lower limit of the SW spectrum, the DM hybridizes with the lowest nonuniform VM and, therefore, delocalizes and disappears in the "volume" SW spectrum. Note that the DM does not interact with the lowest spatially uniform VM due to the orthogonality of their polarizations. Far from the hybridization region and for a linearly polarized microwave field the intensity of the defect mode in the FMR spectrum (relative to the main FMR peak) is proportional to the density of the "defect" dots. Obviously, by using right (left) circular polarization of the microwave signal one can suppress DM (fundamental VM), respectively. In the hybridization process the DM intensity tends to become

similar to the intensity of the mode, which it hybridizes with (see Fig. 2b)

The profile of the DM can be obtained as a profile of a forced oscillation of magnetization caused by the eigen-oscillation in the defect dot. Indeed, the magnitude of the magnetization oscillation in the "defect" dot is much greater than in the other dots (see Fig. 1c), and the dynamic influence of the "defect" dot on the other dots plays the dominant role in the formation of the DM. Using the perturbation theory for a nanodots' array developed in [Verba 2012], one can derive the following expression for the DM profile:

$$\frac{|\boldsymbol{m}_j|}{|\boldsymbol{m}_d|} \approx \frac{\left| N^{xx}(\boldsymbol{r}_{jd}) - 2iN^{xy}(\boldsymbol{r}_{jd}) - N^{yy}(\boldsymbol{r}_{jd}) \right|}{2\left| \omega_{DM} - \omega_j \right|} , \qquad (3)$$

where $\boldsymbol{r}_{jd}$ is the distance vector between the j-th dot and the "defect" "d" dot, and ω_j is eigenfrequency of the j-th dot in a position-dependent static magnetic field created by all the other dots. The equation (3) gives a good estimation of the DM profile in all the cases, except the case when the DM is hybridized with WMs. For sufficiently large interdot distances the mutual demagnetization tensor of dots scales as $\hat{N}(\boldsymbol{r}) \sim |\boldsymbol{r}|^{-3}$ [Beleggia 2004]. Therefore, the DM has a power-law localization caused by the long-range magneto-dipole interaction, and when the DM frequency approaches the spectral region of "volume" SW modes $\omega_{DM} \to \omega_j$ the DM is delocalized. Note, also, that the frequency ω_j only weakly depends on the distance to the "defect" dot, so by experimental measurement of the DM profile (for example, using the FMR force microscopy [Kakazei 2008]) one can obtain a map of dynamical components of the mutual demagnetization tensor $\hat{N}(\boldsymbol{r})$.

As one can see from Fig. 2a, the spectral position of WMs relative to the "volume" SW modes, remains the same at any external bias field: the WMs are always situated in the vicinity of the upper edge of the "volume" SW spectrum. This is natural, since both WMs and VMs have vanishingly small amplitudes at the location of a "defect" dot, in which the internal magnetic field depends on the externally applied bias magnetic field B_e differently, compared to all the other dots. Therefore, in a conventional FMR experiment performed on dot arrays with rare point defects of the magnetic order all the spectral range of the "volume" SW modes becomes visible: the lower edge is identified by the fundamental VM, while the upper edge in marked by the spectral position of the WMs.

III. "MATERIAL" DEFECT

Now, we shall consider a point defect caused by the variation of the dots' material parameters. To give a simple example of such a "material" defect we shall assume that one of the dots in a dot array existing in the FM stationary state has a value of the perpendicular uniaxial anisotropy (in the z-direction), that is different from the value of the same parameter in all the other dots. This additional anisotropy of a single "defect" dot leads to the renormalization of the zz component of the self demagnetization tensor of this dot $N_s^{zz} \to N_s^{zz} - H_{an}/M_s$, which results in the increase of the static internal field an the location of the "defect" dot by the value of the additional anisotropy field H_{an}. All the other characteristics of the FM dot array are not affected.

FIG 3 HERE

The FMR absorption spectrum of an array with a "material" ("anisotropic") point defect is shown in Fig. 3a. As one can see, there are only two absorption peaks, associated with the uniform VM and the "defect" eigenmode (DM), respectively. The DM has a profile similar to the one shown in Fig. 1c: the DM is strongly localized and the localization obeys a power-law. Since the "anisotropic" defect doesn't produce an inhomogeneity of the static internal magnetic field, the "well" modes are absent. The absence of the field "well" together with the identical polarizations of the oscillations in the "defect" dot and in all the other dots leads to a very weak scattering of the VMs on the defect. Therefore, the nonuniform VMs have a small excitation efficiency, and become practically invisible in the FMR spectrum.

For the positive values of H_{an} (easy-axis anisotropy) the DM is situated above the "volume" SW spectrum, while for the negative values of H_{an} (easy-plane anisotropy) the DM is below the "volume" SW spectrum (see Fig. 3b). When the DM frequency approaches the "volume" SW spectrum, the DM hybridizes with the uniform ($H_{an} < 0$) or nonuniform ($H_{an} > 0$) VMs. As in the case of the "magnetic order" defect, this hybridization leads to DM delocalization, and its disappearance in the "volume" SW spectrum. Also, the excitation efficiency of the DM tends to become similar to the intensity of the VM, with which the DM hybridizes (see Fig. 3c). If the DM is situated it the below VM spectrum, its intensity is non small, but its frequency is close to the FMR frequency of an ideal dot array. On the other hand, if the DM lies above VM spectrum, its frequency differs significantly from the ω_{FMR}, but its intensity is rather small. Therefore, the low-contrast "material" defects lead only to a relatively weak broadening of the FMR line, in drastic contrast to the defects of the magnetization order, which may lead to a significant broadening of FMR line due to the existence of the WMs.

IV. CONCLUSION

In conclusion, we have demonstrated, that a single point defect in a magnetic dot array may lead to the appearance of several localized modes – one "defect" mode, which is always present, and several "well" modes, which appear if the "defect" dot creates a sufficiently large spatial inhomogeneity of the internal static magnetic field in the array. The number and the structure of all the localized modes strongly depend on the magnetostatic interaction between the dots. Therefore, the investigation of these "defect" modes can provide valuable

information about the interdot magneto-dipolar interaction. Also, in the presence of a point defect of the magnetization order all the spectral range of the "volume" SWs becomes visible in the FMR absorption spectrum of the array.

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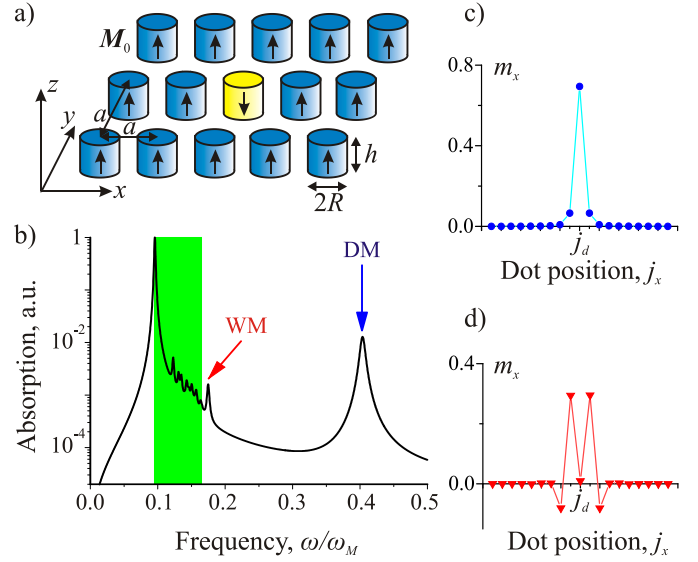
Fig. 1. (a) Sketch of a dot array containing one point defect of the magnetic order – a dot with inverted magnetization. (b) FMR absorption spectrum in a zero external field of a dot array (10×10 dots) containing a single point defect Fig.1a.; the green area indicates the spectral range of "volume" SW modes in the array; (c) and (d) – profiles $m_x(j)$ of the "defect" SW mode (DM) and the most intensive "well" SW mode (WM), respectively. Polarization of the microwave field is linear.
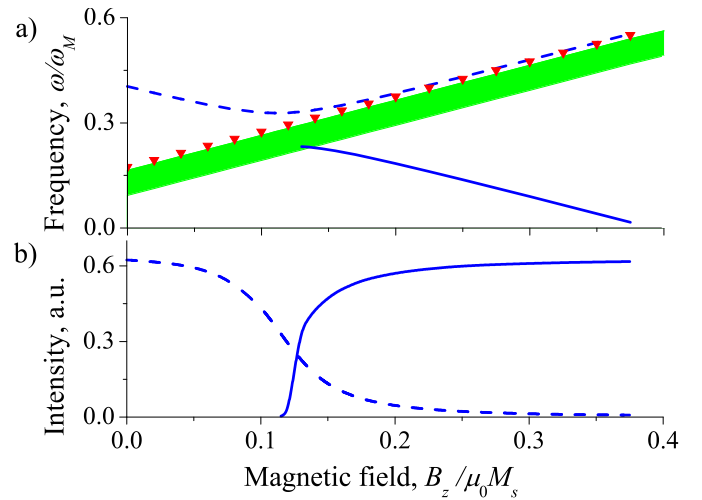

Fig. 2. (a) Frequencies of DM (blue solid and dashed lines), WMs (red triangles) and VMs (green filled area) as functions of the axial bias magnetic field; (b) Excitation efficiency of DM during the hybridization with the lowest non-uniform VM (solid line) and WM (dashed line).
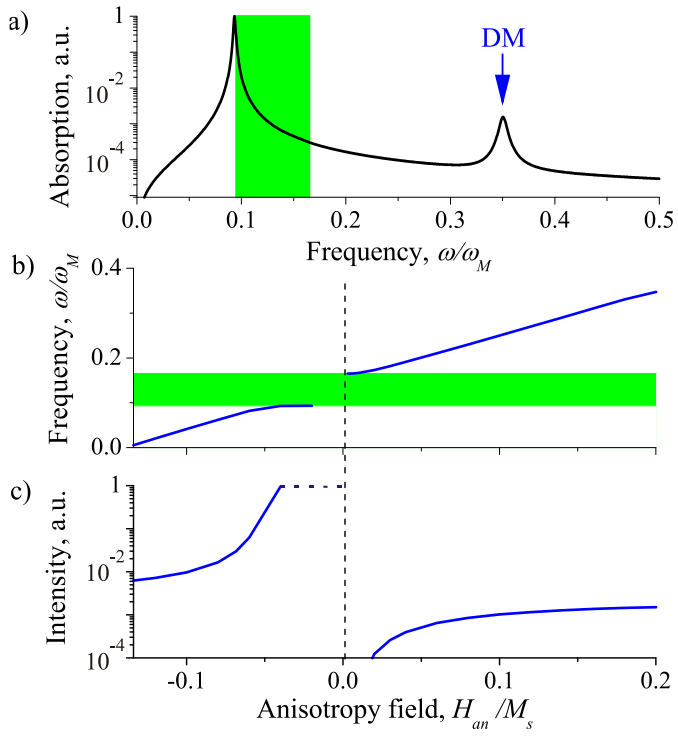
Fig. 3. (a) FMR absorption spectrum of an array having one "material" (anisotropy) defect per 10×10 dots ($H_{an}/M_s = 0.2$); green area indicates the range of the VMs; (b) and (c) – frequency and excitation efficiency of the DM, respectively as functions of the magnitude of the defect anisotropy.